GPS radio occultation ionospheric profile fitting and data quality examination using the empirical orthogonal function analysis

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FormoSat-3 (FS3) /COSMIC: 
~ 2500 radio occultation (RO) measurements worldwide daily

Numbers of daily RO observations on board FS3 / COSMIC and retrieved $N_e$ profiles
Top level view of FS3/COSMIC ionospheric data processing

UCAR LEVEL 1 PROCESSING

IGS fiducial and orbital data
COSMIC Science Data
High Rate fiducial data

Level 1 Processing
Precision Orbit Determination Processing
Precise Orbit Determination Processing
Abel Inversion Processin

TEC Excess Phase files

NCU LEVEL 2/3 PROCESSING

Compensated TEC
2D/3D Initial Guess
MART Algorithm

Vertical Ne Profiles
2D/3D Tomography
foF2, hmF2, foE, and hmE numerical maps
3D/4D Ne images

2D/3D Red-Black Smoothing
Spherical Harmonics Processing
TWIM Modelling

N_e Profile Quality Examination
GPS RO TEC observations and vertical $N_e$ retrieval

Assuming locally spherical symmetry of the ionospheric $N_e$ and straight-line propagation for the GPS L1 and L2 signals:

1. **Calibrated TEC determination (TEC’)**

$$ TEC'(r_t) = pTEC(P_1(r_t)) - pTEC(P_2(r_t)) \approx 2 \int_{r_t}^{r_{LEO}} \frac{r N_e(r)}{\sqrt{r^2 - r_t^2}} \, dr . $$

where $pTECs$ are the path TECs from GPS to LEO, and P1 and P2 are the occulting and auxiliary positions during a GPS RO observation.

2. **The Abel inversion through calibrated TECs**

$$ N_e(r_t) = -\frac{1}{\pi} \int_{r_t}^{r_{LEO}} \frac{d TEC'(r)}{\sqrt{r^2 - r_t^2}} \, dr . $$

Example calibrated TECs (in green) and resulting $N_e$s (in red)
Improved Abel inversion based on compensated TECs

• Compensated TEC determination (TEC*):

\[ TEC^*(r) = TEC'(r) + \int_{P_1}^{P_2} \left( N_{e\text{ model}}(r, \theta_t(r), \lambda_t(r)) - N_{e\text{ model}}(r, \theta, \lambda) \right) dl \]

where \((r, \theta_t(r), \lambda_t(r))\) is the corresponding tangent-point position with the same radial distance along a ray.

• The Abel inversion through compensated TECs:

\[ N_{e\text{ *}}(r_t) = -\frac{1}{\pi} \int_{r_i}^{r_{LEO}} \frac{d TEC^*(r)}{d r} d r \]
$N_e$ profile quality examination by EOF analysis

- Vertical Empirical Orthogonal Function (EOF) fitting of $N_e$ profiles

$$
N_e(h/h_m F_2, \theta, \lambda) = N_{e \text{ max}} \times \left( \frac{N_e(h/h_m F_2, \theta, \lambda)}{N_{e \text{ max}}} \right) + \sum_{i=1}^{n} a_i \times EOF_i(h/h_m F_2)
$$

where $N_e(\ )$ is the peak-height-normalized distribution of $N_e$ profile, EOF(\ )s are the derived empirical orthogonal functions, and $a_i$ is the weighting coefficient for the $i$th retrieved EOF function.

Comparisons between EOF fitting profiles (in blue) and RO $N_e$ profiles (in red)
The mean and normalized $N_e$ profile and the first ten empirical orthogonal functions after being normalized in $N_e$ and a peak density height of 290 km:
Classification of retrieved RO $N_e$ profiles

1. Normal $N_e$ profiles (Examples & statistical EOF coefficient means and deviations, and fitted $N_e$ difference mean and deviation)
2. Abnormal $N_e$ profiles (including a sporadic E layer)

3. Abnormal $N_e$ profiles (including spread features)
3. Abnormal $N_e$ profiles (including strong E layer)

4. Abnormal $N_e$ profiles (including strong F1 layer)
Modelling of 3D Ionospheric Electron Density: The TaiWan Ionospheric Model (TWIM)

- Approach: vertical $\alpha$-Chapman layers + 2D surface spherical harmonics fitting

$$N_e(\theta, \lambda, h) = \sum_{i=1}^{n} N_{e \text{max}}(\theta, \lambda) \times e^{\frac{1}{2} \left\{ 1 - \frac{h-h_m(\theta, \lambda)}{H(\theta, \lambda)} - e^{\frac{h-h_m(\theta, \lambda)}{H(\theta, \lambda)}} \right\}}$$

where $n=6$ for $F_2\text{top}$, $F_2\text{bottom}$, $F1.5$, $F1$, $E$, and $D$-layers, and the peak density ($N_{e \text{max}}$), peak height ($h_m$), and scale height ($H$) are combinations of surface spherical harmonics.

- Advantage: equivalent layers for physical layers; peak height and peak density for each physical layer are explicit parameters.
  Drawback: inaccuracy of fitting profiles.

Numerical mapping of ionospheric parameters (e.g. $f_0F_2$ and $h_mF_2$) by the least squares method

- The spherical surface Laplace equation is
  \[
  \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \Gamma}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 \Gamma}{\partial \phi^2} = 0.
  \]

- The resulting real and orthogonal functions are defined by

\[
U_{nm}(\theta, \phi) = \sqrt{\frac{2n+1}{2\pi} \frac{(n-m)!}{(n+m)!}} \ P_n^m(\cos \theta) \cos m\phi, \text{ and}
\]

\[
V_{nm}(\theta, \phi) = \sqrt{\frac{2n+1}{2\pi} \frac{(n-m)!}{(n+m)!}} \ P_n^m(\cos \theta) \sin m\phi, \text{ where } n = 0, 1, 2, \ldots, m = 0, 1, 2, \ldots, n.
\]

- Includes universal time (UT) and local time (LT) modes.
TWIM Application I: 3D ray tracing

One central ray (in white) and four side rays (in red and blue)

Global top view on $foF2$ map

Latitudinal side view in $fn$ map

Regional top view on $foF2$ map

Longitudinal side view in $fn$ map
Application II: GPS-L1 ionospheric correction

1130 UT
01 Oct 2010

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Validation of the TWIM $foF2$ and $foE$ values (including abnormal $N_e$ profiles) using ionosonde data

- Evaluated by ionosonde data (49 stations) from days 81~180 in 2008.
- ~4000 matches out of ~150,000 RO observations.

- mean (rms) $foF2$ difference = -0.12 (0.76) MHz
- mean (rms) $foE$ difference = -0.37 (0.94) MHz
- The mean $dfoF2$s are $\leq 0.5$ MHz averagely and $\leq 0.25$ MHz from 0° to 60° dip-latitude region.
- The retrieved $foE$ values are underestimated and typically worse at the southern hemisphere.
Validation of the TWIM foF2 and foE values (including abnormal $N_e$ profiles) using ionosonde data

- The mean $dfoEs$ are improved significantly from 0.37 to 0.07 MHz, and the rms $dfoEs$ are also improved from 0.94 to 0.54 MHz.
- The mean and rms $dfoF2s$ are also improved but slightly from -0.12 to -0.11 MHz and from 0.76 to 0.70 MHz, respectively.
- The mean $dfoF2s$ are $\leq 0.5$ MHz averagely and $\leq 0.25$ MHz from 0° to 60° dip-latitude region.
Final remarks

- FormoSat-3/COSMIC has become a promising program for monitoring the large-scale global ionosphere.

- A 3D approach using vertical $\alpha$-type Chapman layers and surface spherical harmonics for modeling variations of ionospheric $N_e$ has been proposed and successfully implemented to be the TaiWan Ionospheric Model (TWIM).

- Different approaches of $N_e$ profile fitting based on EOF has been proposed and successfully implemented to classify abnormal $N_e$ profiles and to derive better 3D $N_e$ model.